

Stability of Semiclassical Gravity Solutions With Respect to Quantum Metric Fluctuations

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We discuss the stability of semiclassical gravity solutions with respect to small quantum corrections by considering the quantum fluctuations of the metric perturbations around the semiclassical solution. We call the attention to the role played by the symmetrized 2-point quantum correlation function for the metric perturbations, which can be naturally decomposed into two separate contributions: intrinsic and induced fluctuations. We show that traditional criteria on the stability of semiclassical gravity are incomplete because these criteria based on the linearized semiclassical Einstein equation can only provide information on the expectation value and the intrinsic fluctuations of the metric perturbations. By contrast, the framework of stochastic semiclassical gravity provides a more complete and accurate criterion because it contains information on the induced fluctuations as well. The Einstein–Langevin equation therein contains a stochastic source characterized by the noise kernel (the symmetrized 2-point quantum correlation function of the stress tensor operator) and yields stochastic correlation functions for the metric perturbations which agree, to leading order in the large N limit, with the quantum correlation functions of the theory of gravity interacting with N matter fields. These points are illustrated with the example of Minkowski space-time as a solution to the semiclassical Einstein equation, which is found to be stable under both intrinsic and induced fluctuations.

KEY WORDS: semiclassical gravity; quantum metric fluctuations; stochastic gravity; Einstein–Langevin equation.

1. INTRODUCTION

In this paper we discuss the stability of the solutions of semiclassical gravity (SCG) (Birrell and Davies, 1994; Fischetti *et al.*, 1979; Flanagan and Wald, 1996; Hartle and Hu, 1979; Hu and Parker, 1978; Wald, 1994; Zeldovich and Starobinsky, 1972) emphasizing the role of metric fluctuations induced by the quantum matter sources. SCG is based on the self-consistent solutions of the semiclassical Einstein

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equation for a classical space-time driven by the expectation value of the stress tensor operator of quantum matter fields. We propose a criterion based on stochastic semiclassical gravity which involves the fluctuations of the metric.

SCG accounts for the averaged back reaction of quantum matter fields and can be regarded as a mean field approximation that describes the dynamics of the mean space-time geometry. However, it does not account for the effects of the fluctuations of space-time geometry. Here we focus on the effects of the quantum fluctuations of the metric. We will restrict our treatment to small metric perturbations around a given background geometry. One can then use the stochastic semiclassical gravity formalism (Hu and Verdaguer, 2003, 2004) to study the fluctuations of the metric perturbations. In fact, one can show that the leading order contribution to the quantum correlation functions in a large N expansion is equivalent to the stochastic correlation functions obtained by solving the Einstein–Langevin equation in the context of stochastic semiclassical gravity. By *leading order* in the large N limit we mean the lowest order in $1/N$ with a nonvanishing contribution (thus, when using the rescaled gravitational coupling constant introduced in Section 2.2, the leading order for the source of the semiclassical Einstein equation is $1/N^0$, whereas the leading order for the quantum 2-point correlation function is $1/N$).

Making use of the equivalence between quantum and stochastic correlation functions in stochastic semiclassical gravity, one is naturally led to separate the symmetrized quantum correlation function for the metric perturbations (to leading order in $1/N$) into two separate contributions: the *intrinsic* and the *induced* fluctuations. The former is connected to the dispersion of the initial state of the metric perturbations, whereas the latter is induced by the quantum fluctuations of the matter fields' stress tensor operator.

Different aspects concerning the validity of the description provided by SCG in the case of free quantum matter fields in the Minkowski vacuum state propagating on Minkowski space-time have been studied by a number of authors. Most of them considered the stability of such a solution of SCG with respect to small perturbations of the metric. Horowitz was the first one to analyze the equations describing those perturbations, which involve higher order derivatives (up to fourth order), and found unstable solutions that grow exponentially with characteristic timescales comparable to the Planck time (Horowitz, 1980, 1981). This was later reanalyzed by Jordan with similar conclusions (Jordan, 1987). However, those unstable solutions were regarded as an unphysical artifact by Simon, who argued that they lie beyond the expected domain of validity of the theory and emphasized that only those solutions which resulted from truncating the perturbative expansions in terms of the square of the Planck length are acceptable (Simon, 1990, 1991). Further discussion was provided by Flanagan and Wald (1996), who advocated the use of an order reduction prescription first introduced by Parker and Simon (1993) but insisted that even nonperturbative solutions of the resulting second-order equation should be regarded as acceptable. Following these approaches,

Minkowski space-time is shown to be a stable solution of SCG with respect to small metric perturbations.

Anderson, Molina-París, and Mottola have recently taken up the issue of the validity of SCG (Anderson *et al.*, 2003) again. Their starting point is the fact that the semiclassical Einstein equation will fail to provide a valid description of the dynamics of the mean space-time geometry whenever the higher order radiative corrections to the effective action, involving loops of gravitons or internal graviton propagators, become important (see Tsamis and Woodard, 1996 a,b, 1997, 1998, for some attempts to include those effects). Next, they argue qualitatively that such higher order radiative corrections cannot be neglected if the metric fluctuations grow without bound. Finally, they propose a criterion (a necessary condition) to characterize the growth of the metric fluctuations, and hence the validity of SCG, based on the stability of the solutions of the linearized semiclassical equation.

This is a summary of a recent paper we wrote (Hu *et al.*, 2004) addressing the issue of the stability of semiclassical solutions with respect to small quantum corrections. When the metric perturbations are quantized, the semiclassical equation can be interpreted as the equation governing the evolution of the expectation value of the operator for the metric perturbations. We introduce a stability criterion based on whether the metric fluctuations grow without bound or not by considering the behavior of the quantum correlation functions of the metric perturbations. We emphasize that one should consider not only the intrinsic fluctuations but also the induced ones. It is true that the effect of intrinsic fluctuations can be deduced from an analysis of the solutions of the perturbed semiclassical Einstein equation, but in general one cannot retrieve the effect of the induced fluctuations from it. This effect can be properly accounted for in the stochastic semiclassical gravity framework. Both intrinsic and induced fluctuations are innate in the Einstein-Langevin equation.

Throughout the paper we use natural units with $\hbar = c = 1$ and the (+, +, +) convention of Misner *et al.* (1973). We also make use of the abstract index notation of Wald (1984). Latin indices denote abstract indices, whereas Greek indices are employed when a particular coordinate system is considered.

2. SEMICLASSICAL GRAVITY AND STOCHASTIC SEMICLASSICAL GRAVITY

2.1. Semiclassical Gravity

A possible first step when addressing the interplay between gravity and quantum field theory is to consider the evolution of quantum matter fields (matter field is referred to here as any field other than the gravitational one) on a classical space-time with a nontrivial geometry, characterized by a metric g_{ab} . As opposed to the situation for a Minkowski space-time, there is in general no preferred vacuum

state for the fields and particle creation effects naturally arise, such as Hawking radiation for black holes, cosmological particle creation, and the generation of primordial inhomogeneities in inflationary cosmological models. *Quantum field theory in curved space-time* (QFTCST) is by now a well-established subject (at least for free fields and globally hyperbolic space-times). (Birrell and Davies, 1994; Wald, 1994).

QFTCST is only an approximation in that the matter fields are treated as test fields evolving on a given space-time. Einstein's theory requires that space-time dynamics determines and is determined by the matter field. Thus one needs to consider the back reaction of the quantum matter fields on the dynamics of the space-time geometry, which naturally leads to the *semiclassical theory of gravity*, where the evolution of the space-time metric g_{ab} is determined by the semiclassical Einstein equation

$$G_{ab}[g] + \Lambda g_{ab} - \alpha A_{ab}[g] - \beta B_{ab}[g] = \kappa \langle \hat{T}_{ab}[g] \rangle'_{\text{ren}}, \quad (1)$$

where g_{ab} is the space-time metric, $G_{ab}[g]$ is the Einstein tensor, and the matter source corresponds to the renormalized expectation value of the stress tensor operator of the matter fields (a prime was used to distinguish it from that introduced below after absorbing some terms). Here, Λ is the renormalized cosmological constant, $\kappa = 8\pi G$, with $G \equiv 1/m_p^2$ being the Newton constant and m_p the Planck mass. α and β are renormalized dimensionless coupling constants associated with tensors $A_{ab}[g]$, $B_{ab}[g]$ needed for the renormalization of the logarithmic divergences (the renormalized coupling constants are running coupling constants which depend on some renormalization scale μ ; however, since $\langle \hat{T}_{ab}[g] \rangle'_{\text{ren}}$ has the same dependence on μ , the semiclassical Einstein equation is invariant under the renormalization group, which involves changes in the renormalization scale μ). The expectation value of the stress tensor operator exhibits divergences which are local and state independent. Introducing a covariant regularization and renormalization procedure, those divergences can be absorbed into the cosmological constant, the Newton constant multiplying the Einstein–Hilbert term and the gravitational action counterterms quadratic in the curvature. The finite contributions from those counterterms give rise to the covariantly conserved tensors A_{ab} and B_{ab} which result from functionally differentiating with respect to the metric the terms $\int d^4x \sqrt{-g} C^{abcd} C_{abcd}$ and $\int d^4x \sqrt{-g} R^2$, respectively, where C_{abcd} is the Weyl tensor and R is the Ricci scalar. Those contributions were explicitly written on the left-hand side of Eq. (1), but from now on will be included in the renormalized expectation value of the stress tensor operator so that the semiclassical Einstein equation becomes

$$G_{ab}[g] = \kappa \langle \hat{T}_{ab}[g] \rangle_{\text{ren}}. \quad (2)$$

The field operators appearing in the stress tensor operator for the quantum matter fields are in the Heisenberg picture and satisfy the corresponding equation of

motion, which coincides with the classical field equation for fields evolving on that space-time. In particular, if we consider a free scalar field, the field operator in the Heisenberg picture will satisfy the corresponding Klein–Gordon equation for that geometry.

Given a manifold \mathcal{M} and a metric g_{ab} , which characterize a globally hyperbolic space-time, and a density matrix $\hat{\rho}$, which specifies the state of the quantum matter fields on a particular Cauchy hypersurface, the triplet $(\mathcal{M}, g_{ab}, \hat{\rho})$ constitutes a solution of SCG if it is a self-consistent solution of both the semiclassical Einstein Eq. (2) and the equations of motion for the quantum operators of the matter fields evolving on the space-time manifold \mathcal{M} with metric g_{ab} . Those operators enter in turn into the definition of the *stress tensor operator* appearing in the semiclassical Einstein equation.

One can always consider small metric perturbations around a given solution of semiclassical gravity characterized by a metric g_{ab} . The linearized semiclassical equation for the metric perturbations becomes then

$$G_{ab}^{(1)}[g + h] = \kappa \langle \hat{T}_{ab}^{(1)}[g + h] \rangle_{\text{ren}}, \quad (3)$$

where the superindex (1) was used to denote that only terms linear in the metric perturbation h_{ab} should be considered. The expectation value $\langle \hat{T}_{ab}^{(1)}[g + h] \rangle_{\text{ren}}$ can be evaluated working directly with the quantum operators for the matter fields in the Heisenberg picture in some cases (Martín and Verdaguer, 1999), but is usually more convenient to obtain it from the corresponding effective action in the CTP formalism (Calzetta and Hu, 1987, Campos and Verdaguer, 1994; Jordan, 1986).

2.2. Stochastic Semiclassical Gravity

The semiclassical Einstein equation, which takes into account only, the mean values, is inadequate whenever the fluctuations of the stress tensor operator are important. An improved treatment is provided by the Einstein–Langevin equation of *stochastic gravity*, which contains a (Gaussian) stochastic source with a vanishing expectation value and a correlation function characterized by the symmetrized 2-point function of the stress tensor operator. This theory has been discussed by a number of authors (Calzetta *et al.*, 1997; Calzetta and Hu, 1994; Campos and Verdaguer, 1996; Hu and Matacz, 1995; Hu and Sinha, 1995; Hu and Verdaguer, 2003, 2004; Martín and Verdaguer, 1999). Consider a globally hyperbolic background space-time and an initial state for the quantum matter fields (one usually restricts to free fields) which constitute a self-consistent solution of SCG, i.e., they satisfy the semiclassical Einstein equation with the expectation value of the stress tensor operator obtained by considering the evolution of the matter fields on the same background geometry. The Einstein–Langevin equation governing the dynamics of the linearized perturbations h_{ab} around the background metric g_{ab} is

given by

$$G_{ab}^{(1)}[g+h] = \kappa \langle \hat{T}_{ab}^{(1)}[g+h] \rangle_{\text{ren}} + \kappa \xi_{ab}[g], \quad (4)$$

where the Gaussian stochastic source $\xi_{ab}[g]$ is completely characterized by its correlation function in terms of the noise kernel $\mathcal{N}_{abcd}(x, y)$, which accounts for the fluctuations of the stress tensor operator, as follows:

$$\langle \xi_{ab}[g; x] \xi_{cd}[g; y] \rangle_{\xi} = \mathcal{N}_{abcd}(x, y) \equiv \frac{1}{2} \langle \{ \hat{t}_{ab}[g; x], \hat{t}_{cd}[g; y] \} \rangle, \quad (5)$$

where $\hat{t}_{ab} \equiv \hat{T}_{ab} - \langle \hat{T}_{ab} \rangle$ and $\langle \dots \rangle$ is the usual expectation value with respect to the quantum state of the matter fields, whereas $\langle \dots \rangle_{\xi}$ denotes taking the average with respect to all possible realizations of the stochastic source ξ_{ab} . Note that any local term quadratic in the curvature arising from finite contributions of the counterterms required to renormalize the bare expectation value of the stress tensor operator has been absorbed into its renormalized version $\langle \hat{T}_{ab}^{(1)}[g+h] \rangle_{\text{ren}}$. It should also be emphasized that the solutions of the Einstein–Langevin equation for the metric perturbations are classical *stochastic* tensorial fields, not quantum operators.

The precise meaning that should be given to these stochastic metric perturbations and the relation of the corresponding stochastic correlation functions to the quantum fluctuations that result from quantizing these metric perturbations can be established by considering N matter fields. Making use of a large N expansion, one can then show that the stochastic correlation functions for the metric perturbations obtained from the Einstein–Langevin equation coincide with the leading order contribution to the quantum correlation functions in the large N limit (Hu *et al.*, 2004; Roura and Verdaguer, manuscript in preparation-a). In particular the 2-point stochastic correlation function is equivalent to the symmetrized quantum correlation function to leading order in $1/N$ provided that one also averages over the initial conditions for the solutions of the Einstein–Langevin equation distributed according to the Wigner functional characterizing the initial state of the metric perturbations (see Eq. (C11) in Hu *et al.*, 2004, for the definition of the Wigner functional). It is, therefore, convenient to express the solutions of the Einstein–Langevin equation as

$$h_{ab}(x) = \Sigma_{ab}^{(0)}(x) + \bar{\kappa} (G_{\text{ret}} \cdot \xi)_{ab}(x), \quad (6)$$

where we have introduced the notation $A \cdot B \equiv \int d^4y \sqrt{-g(y)} A(y) B(y)$, $\bar{\kappa} = N\kappa$ is the rescaled gravitational coupling constant introduced in Hu *et al.* (2004), $\Sigma_{ab}^{(0)}(x)$ is a solution of the homogeneous part of the Einstein–Langevin equation (4) containing all the information about the initial conditions (by homogeneous part we mean Eq. (4) excluding the stochastic source, which coincides with the semiclassical Einstein equation (2)), and $G_{\text{ret}}(x, x')$ is the retarded propagator with vanishing initial conditions associated with that equation (see Appendix E.3 in Hu *et al.* (2004) for important remarks on the propagator). Using Eq. (5), we

can then get the following result for the symmetrized 2-point quantum correlation function of the metric perturbations around a Minkowski background:

$$\frac{1}{2} \langle \{ \hat{h}_{ab}(x), \hat{h}_{cd}(x') \} \rangle = \left\langle \Sigma_{ab}^{(0)}(x) \Sigma_{cd}^{(0)}(x') \right\rangle_{\Sigma_{ab}^{(i)}(x), \Pi_{\hat{g}}^{cd}} + \frac{\bar{\kappa}^2}{N} (G_{\text{ret}} \cdot \mathcal{N} \cdot (G_{\text{ret}})^T)_{abcd}(x, x'), \quad (7)$$

where the Lorentz gauge condition $\nabla^a (h_{ab} - (1/2)\eta_{ab}h^c_c) = 0$ as well as some initial condition to fix completely the remaining gauge freedom of the initial state should be implicitly understood, and the stochastic source was rescaled according to Hu *et al.* (2004) and Roura and Verdaguier (manuscript in preparation-a) so that $\langle \xi_{ab}[g; x] \xi_{cd}[g; y] \rangle_\xi = (1/N) \mathcal{N}_{abcd}(x, y)$, where $\mathcal{N}_{abcd}(x, y)$ is the noise kernel for a single field.

There are two different contributions to the symmetrized quantum correlation function. The first one is connected to the quantum fluctuations of the initial state of the metric perturbations and we will refer to it as *intrinsic fluctuations*. The second contribution, proportional to the noise kernel, accounts for the fluctuations due to the interaction with the matter fields, and we will refer to it as *induced fluctuations*.

3. STABILITY OF SEMICLASSICAL GRAVITY SOLUTIONS: PREVIOUS WORK

Although the stability of other semiclassical gravity solutions in addition to Minkowski space-time has been studied (see, for instance, Starobinsky, 1980; Suen, 1985; Vilenkin, 1985, for analysis involving Robertson–Walker geometries), most of the analyses have concentrated on the stability of small perturbations around Minkowski space-time. This case already exhibits the main features and difficulties that one may encounter when dealing with back-reaction effects in semiclassical gravity and will be used in the next section to illustrate the generalized stability criterion introduced there. In this section we give a brief review of previous work on the stability of semiclassical gravity solutions specialized, for the reasons mentioned above, to the case of Minkowski space-time.

The stability of metric perturbations around a Minkowski space-time interacting with quantum matter fields in their Minkowski vacuum state was first studied in the context of SCG by Horowitz (1980). He considered massless conformally coupled scalar fields and found exponential instabilities for the linearized metric perturbations with characteristic timescales comparable to the Planck time. Those solutions are closely related to the higher derivative counterterms required to renormalize the expectation value of the stress tensor operator and are analogous to the runaway solutions commonly present in radiation reaction processes

such as those considered in classical electrodynamics (Jackson, 1999; Johnson and Hu, 2002). It is generally believed that the runaway solutions obtained by Horowitz are an unphysical artifact since they involve scales beyond the regime where SCG is expected to be reliable (in fact, this statement can be naturally formulated when regarding general relativity as a low-energy effective theory (Burgess, 2003).

Since the existence of terms with higher derivatives in time implies an increase in the number of degrees of freedom (in an initial value formulation, not only the metric and its time derivative should be specified, but also its second- and third-order time derivatives), it seems plausible that, by restricting to an appropriate subspace of solutions of the semiclassical Einstein equation, one can reestablish the usual number of degrees of freedom in general relativity and, at the same time, get rid of all the unphysical runaway solutions. Following this line of thought, Simon proposed that one should restrict to solutions which result from truncating to order \hbar an analytic expansion in \hbar (or equivalently in l_p^2 , the Planck length squared) (Simon, 1990, 1991). Together with Parker, he also introduced a prescription to reduce the order of the semiclassical Einstein equation which was computationally convenient for obtaining solutions corresponding to such truncated perturbative expansions in \hbar (Parker and Simon, 1993).

On the other hand, Flanagan and Wald argued that Simon's criterion based on truncating to order \hbar solutions which correspond to analytic expansions in \hbar seemed too restrictive since it only allowed small deviations with respect to the classical solutions of the Einstein equations (Flanagan and Wald, 1996). In particular, one would miss those situations in which the small semiclassical corrections build up to give significant deviations at long times, such as those corresponding to the evaporation of a macroscopic black hole (with a mass much larger than the Planck mass) by emission of Hawking radiation. Furthermore, they illustrated with simple examples that there are cases in which one expects that no solutions of the semiclassical equation are analytic in \hbar . Therefore, they suggested that, rather than trying to restrict the subspace of acceptable solutions, one should simply transform the semiclassical equation, by making use of Simon and Parker's order reduction prescription, to a second-order equation which was equivalent to the original equation up to the order in \hbar (or l_p^2) under consideration. All the solutions of the second-order equation should then be regarded as acceptable, even if they are not analytic in \hbar . Obviously, one could only extract physically reliable information from those solutions for scales much larger than the Planck length.

Yet another prescription was proposed by Anderson *et al.* (2003) on the stability of small metric perturbations around the Minkowski space-time. They got rid of the unphysical runaway solutions by working in Fourier space and discarding those solutions which corresponded to 4 momenta with modulus comparable or larger in absolute value than the Planck mass. However, it is not clear how this

procedure could be generalized to situations where working in Fourier space is not adequate, as in time-dependent background space-times.

The consequences of both the order reduction prescription introduced by Simon and Parker and advocated by Flanagan and Wald, and the procedure employed by Anderson *et al.* are rather drastic, at least when applied to the case of a Minkowski background, since one is just left with the solutions of the sourceless classical Einstein equation corresponding to linear gravitational waves propagating in Minkowski. In fact, the situation was not completely trivial for Flanagan and Wald, who were interested in analyzing whether the averaged null energy condition (ANEC) was satisfied in SCG by considering perturbations of the Minkowski solution, because they also perturbed the state of the matter fields. The order reduction prescription also seems to exclude those solutions which correspond to inflationary models driven entirely by the vacuum polarization of the quantum matter fields (Simon, 1992), such as the trace anomaly-driven inflationary model initially proposed by Starobinsky (1980). To keep this kind of models, Hawking, Hertog, and Reall considered a less drastic alternative to deal with the runaway solutions (Hawking *et al.*, 2001; Hawking and Hertog, 2002). Their procedure, which is analogous to some methods previously employed in classical electrodynamics for radiation reaction problems (Jackson, 1989), is based on discarding solutions which grow without bound at late times (see Hu *et al.*, 2004, for further discussions on this and related issues).

4. GENERALIZED STABILITY CRITERION. APPLICATION TO MINKOWSKI SPACE-TIME

4.1. Generalized Stability Criterion

How does one characterize the quantum state of the metric perturbations? The first candidate is the expectation value of the operator associated with the perturbation of the metric, \hat{h}_{ab} . In fact, using a large N expansion, Hartle and Horowitz showed that the semiclassical Einstein equation can be interpreted as the equation governing the evolution of the expectation value of the metric to leading order in $1/N$ (Hartle and Horowitz, 1981). Taking that result into account, the study of the stability of a solution of SCG by linearizing the semiclassical Einstein equation with respect to small metric perturbations around that solution can be understood in the following way: Take an initial state for the metric perturbations with a small nonvanishing expectation value for the operator \hat{h}_{ab} , let it evolve, and see if the expectation value grows without bound.

However, in addition to the expectation value of \hat{h}_{ab} , the state of the metric perturbations will also be characterized by its fluctuations. Let us now suppose that the evolution of the expectation value is stable (i.e., that it does not grow unboundedly with time) or even that it vanishes for all times. It is clear that the

semiclassical solution cannot be regarded as stable with respect to small quantum corrections if the fluctuations of the state for the metric perturbations grow without bound. Therefore, the stability criteria based on the solutions of the semiclassical Einstein equation, which can be interpreted as conditions on the stability of the expectation value of the operator \hat{h}_{ab} for the state of the metric perturbations, should be generalized: one also needs to take into account the fluctuations. In addition to the expectation value, the n -point quantum correlation functions for the metric perturbations (starting with $n = 2$) should also be stable.

As explained in Hu *et al.* (2004) and Roura and Verdaguer (manuscript in preparation-a), to leading order in $1/N$ the CTP generating functional for the metric perturbations exhibits a Gaussian form provided that a Gaussian initial state for the metric perturbations with vanishing expectation value is chosen. All the n -point quantum correlation functions can then be obtained, to leading order in $1/N$, from the 2-point quantum correlation function. Furthermore, any of the 2-point quantum correlation functions can in turn be expressed in terms of the symmetrized and antisymmetrized correlation functions (the expectation values of the commutator and anticommutator of the operator \hat{h}_{ab}). To leading order in $1/N$ the commutator is independent of the initial state of the metric perturbations and is given by $2i\kappa(G_{\text{ret}}(x, x') - G_{\text{ret}}(x', x))$. On the other hand, the expectation value of the anticommutator is given by Eq. (7) and is the sum of two separate contributions: the intrinsic and the induced fluctuations.

The first contribution in Eq. (7) to the correlation function for the metric perturbations involves the solutions of the homogeneous part of the Einstein–Langevin equation (4), which actually coincides with the linearized semiclassical equation for the metric perturbations around the background geometry. Similarly, G_{ret} corresponds to the retarded propagator (with vanishing initial conditions) associated with the linearized semiclassical equation. Thus, solving the perturbed semiclassical Einstein equation not only accounts for the evolution of the expectation value of the metric perturbations, which will exhibit a nontrivial dynamics as long as we choose an initial state with nonvanishing expectation value, but also provides nontrivial information, even for a state with a vanishing expectation value, about the commutator as well as the intrinsic fluctuations of the metric. This implies that the analysis about the stability of the solutions of SCG can also be used to determine the stability of the metric perturbations with respect to intrinsic fluctuations.

The observation we make here is that the induced fluctuations can be important as well. Both the retarded propagator and the solutions of the linearized semiclassical Einstein equation depend on the expectation value of the commutator of the stress tensor operator on the background geometry and on the imaginary part of its time-ordered 2-point function. However, they do not involve the expectation value of the anticommutator, which drives the induced fluctuations. Furthermore, although the expectation values of the commutator and the anticommutator are related by a fluctuation–dissipation relation in some particular cases (Martín and

Verdaguer, 1999, 2000), that is not true in general and the induced fluctuations need to be explicitly analyzed.

To sum up, when analyzing the stability of a solution of SCG with respect to small quantum corrections, one should also consider the behavior of both the intrinsic and the induced fluctuations of the quantized metric perturbations. Whereas information on the stability of the intrinsic fluctuations can be retrieved from an analysis of the solutions of the perturbed semiclassical Einstein equation, the effect of the induced fluctuations is properly accounted for only in the stochastic semiclassical gravity framework based on the Einstein–Langevin equation.

4.2. Stability of Minkowski Space From Our Criterion

We now turn to the application of the criterion proposed in the previous subsection to the particular yet important case of Minkowski space-time. As explained there, the existing results in the literature can be interpreted as analysis of the stability of the expectation value of the operator associated with the metric perturbations (see, however, Jaekel and Reynaud, 1995; Hartle and Horowitz, 1981; Horowitz, 1981). On the other hand, we also need to include in our consideration the fluctuations, characterized by the 2-point quantum correlation function.

To analyze the 2-point quantum correlation function for the metric perturbations, we will exploit the fact that the stochastic correlation functions obtained with the solutions of the Einstein–Langevin equation coincide with the quantum correlation functions for the metric perturbations. Moreover, according to Eq. (7), the symmetrized 2-point quantum correlation function has two different contributions: the intrinsic and the induced fluctuations. We proceed now to analyze each contribution separately.

The first term on the right-hand side of Eq. (7) corresponds to the fluctuations of the metric perturbations due to the fluctuations of their initial state and is given by

$$\left\langle \Sigma_{ab}^{(0)}(x) \Sigma_{cd}^{(0)}(x') \right\rangle_{\Sigma_{ab}^{(i)}, \Pi_{(i)}^{cd}}, \tag{8}$$

where we recall that $\Sigma_{ab}^{(0)}(x)$ is a solution of the homogeneous part of the Einstein–Langevin equation (once the Lorentz gauge has been imposed) with the appropriate initial conditions.

As mentioned in Section 2.2 the homogeneous part of the Einstein–Langevin equation actually coincides with the linearized semiclassical Einstein equation (3). Therefore, we can make use of the results derived in the literature (Anderson *et al.*, 2003; Flanagan and Wald, 1996; Horowitz, 1980), which are briefly summarized in Appendix E of Hu *et al.* (2001). As described there, in addition to the solutions with $G_{ab}^{(1)}(x) = 0$, there are other solutions that in Fourier space take the

form $\tilde{G}_{\mu\nu}^{(1)}(p) \propto \delta(p^2 - p_0^2)$ for some particular values of p_0^2 , but they all exhibit exponential instabilities with Planckian characteristic timescales.

To deal with those unstable solutions, one possibility is to employ the order reduction prescription. We are then left only with the solutions which satisfy $\tilde{G}_{\mu\nu}^{(1)}(p) = 0$ (see Hu *et al.*, 2004). The result for the metric perturbations in the gauge introduced above can be obtained by solving for the Einstein tensor in that gauge: $\tilde{G}_{ab}^{(1)}(p) = (1/2)p^2(\tilde{h}_{\mu\nu}(p) - 1/2\eta_{\mu\nu}\tilde{h}_\rho^\rho(p))$. Those solutions for $\tilde{h}_{\mu\nu}(p)$ simply correspond to free linear gravitational waves propagating in Minkowski space-time expressed in the transverse and traceless (TT) gauge. When substituting back into Eq. (8) and averaging over the initial conditions, we simply get the symmetrized quantum correlation function for free gravitons in the TT gauge for the state given by the reduced Wigner function.

A second possibility, proposed in Hawking *et al.* (2001); Hawking and Hertog (2002), is to impose boundary conditions which discard the runaway solutions that grow unboundedly in time and correspond to a special prescription for the integration contour when Fourier transforming back to space-time coordinates (see Appendix E in Hu *et al.*, 2004, for a more detailed discussion). Following that procedure we get, for example, that for a massless conformally coupled scalar field, with appropriate values of the renormalized coupling constants, the intrinsic contribution to the symmetrized quantum correlation function coincides with that of free gravitons plus an extra contribution for the scalar part of the metric perturbations which renders Minkowski space-time stable but plays a crucial role in providing a graceful exit for inflationary models driven by the vacuum polarization of a large number of conformal fields (such a massive scalar field would not be in conflict with present observations because, for the range of parameters usually considered, the mass would be far too large to have observational consequences; Hawking *et al.*, 2001).

The second term on the right-hand side of Eq. (7) corresponds to the fluctuations of the metric perturbations induced by the fluctuations of the quantum matter fields and is given by

$$\frac{\bar{\kappa}^2}{N}(G_{\text{ret}} \cdot \mathcal{N} \cdot (G_{\text{ret}})^T)_{abcd}(x, x') = N\kappa^2(G_{\text{ret}} \cdot \mathcal{N} \cdot (G_{\text{ret}})^T)_{abcd}(x, x'), \quad (9)$$

where $\mathcal{N}_{abcd}(x, x')$ is the noise kernel accounting for the fluctuations of the stress tensor operator, and $(G_{\text{ret}})_{abcd}(x, x')$ is the retarded propagator with vanishing initial conditions associated with the integro-differential operator $L_{abcd}(x, x')$ defined as

$$L_{abcd}(x, x') = (1/2)(\eta_{ac}\eta_{bd} - \eta_{ab}\eta_{cd}/2)\square\delta(x - x') + 2\bar{\kappa}H_{abcd}^{(\text{ren})}(x - x') + 2\bar{\kappa}M_{abcd}^{(\text{ren})}(x - x'), \quad (10)$$

where the kernel H corresponds to the sum of the expectation values of the commutator and the imaginary part of the time-ordered product of the stress tensor

operator for the matter fields evaluated on the background geometry, and the kernel M is obtained by functionally differentiating with respect to the metric the expectation value of the stress tensor operator on the background geometry taking into account only its explicit dependence on the metric. See Eqs. (5) and (6) in Hu *et al.* (2004) for the exact definition of both kernels.

The same kind of exponential instabilities as in the runaway solutions of the homogeneous part of the Einstein–Langevin equation (the linearized semiclassical Einstein equation) also arises when computing the retarded propagator G_{ret} . To deal with those instabilities, similar to the case of the intrinsic fluctuations, one possibility is to make use of the order reduction prescription. The Einstein–Langevin equation becomes then $G_{ab}^{(1)} = \kappa \xi_{ab}$. The second possibility, following the proposal of Hawking *et al.*, is to impose boundary conditions which discard the exponentially growing solutions and translate into a special choice of the integration contour when Fourier transforming back to space-time coordinates the expression for the propagator. In fact, it turns out that the propagator which results from adopting that prescription coincides with the propagator that was employed in Martín and Verdaguer (2000). However, it should be emphasized that this propagator is no longer the retarded one since it exhibits causality violations at Planckian scales. A more detailed discussion on all these points can be found in Appendix E of Hu *et al.* (2004).

Following Hu *et al.* (2004) and Martín and Verdaguer (2000), the Einstein–Langevin equation can be entirely written in terms of the linearized Einstein tensor $\tilde{G}_{\mu\nu}^{(1)}(p)$. One can then solve the stochastic equation for $\tilde{G}_{\mu\nu}^{(1)}(p)$ and obtain its correlation function (Hu *et al.*, 2004; Martín and Verdaguer, 2000):

$$\begin{aligned} \langle \tilde{G}_{\mu\nu}^{(1)}(p) \tilde{G}_{\rho\sigma}^{(1)}(p') \rangle_{\xi} &= \bar{\kappa}^2 \tilde{D}_{\mu\nu\alpha\beta}(p) \langle \tilde{\xi}^{\alpha\beta}(p) \tilde{\xi}^{\gamma\delta}(p') \rangle_{\xi} \tilde{D}_{\rho\sigma\gamma\delta}(p') \\ &= \frac{\bar{\kappa}^2}{N} \tilde{D}_{\mu\nu\alpha\beta}(p) \tilde{\mathcal{N}}^{\alpha\beta\gamma\delta}(p) \tilde{D}_{\rho\sigma\gamma\delta}(-p) (2\pi)^4 \delta(p + p'), \end{aligned} \quad (11)$$

where the explicit expressions for the noise kernel $\tilde{\mathcal{N}}^{\alpha\beta\gamma\delta}(p)$ and the propagator $\tilde{D}_{\mu\nu\alpha\beta}(p)$ can be found, respectively, in Appendices B and E of Hu *et al.* (2004). On the other hand, if we make use of the order reduction prescription, we get

$$\langle \tilde{G}_{\mu\nu}^{(1)}(p) \tilde{G}_{\rho\sigma}^{(1)}(p') \rangle_{\xi} = \bar{\kappa}^2 \langle \tilde{\xi}^{\alpha\beta}(p) \tilde{\xi}^{\gamma\delta}(p') \rangle_{\xi} = \frac{\bar{\kappa}^2}{N} \tilde{\mathcal{N}}^{\alpha\beta\gamma\delta}(p) (2\pi)^4 \delta(p + p'). \quad (12)$$

Note that $G_{\mu\nu}^{(1)}(p)$ is gauge invariant when perturbing a Minkowski background because the background tensor $G_{ab}^{(0)}$ vanishes and, hence $\mathcal{L}_{\vec{\zeta}} G_{ab}^{(0)}$ also vanishes for any vector field $\vec{\zeta}$.

Finally, using the expression for the linearized Einstein tensor in the Lorentz gauge, $\tilde{G}_{\mu\nu}^{(1)} = (1/2)p^2 \tilde{h}_{\mu\nu}$ with $\tilde{h}_{\mu\nu} = h_{\mu\nu} - (1/2)\eta_{\mu\nu} h_{\alpha}^{\alpha}$, we obtain the

correlation function for the metric perturbations in that gauge:

$$\langle \tilde{h}_{\mu\nu}(p)\tilde{h}_{\rho\sigma}(p') \rangle_{\xi} = \frac{4\bar{\kappa}^2}{N} \frac{1}{(p^2)^2} \tilde{D}_{\mu\nu\alpha\beta}(p)\tilde{N}^{\alpha\beta\gamma\delta}(p)\tilde{D}_{\rho\sigma\gamma\delta}(-p)(2\pi)^4\delta(p+p'), \quad (13)$$

or

$$\langle \tilde{h}_{\mu\nu}(p)\tilde{h}_{\rho\sigma}(p') \rangle_{\xi} = \frac{4\bar{\kappa}^2}{N} \frac{1}{(p^2)^2} \tilde{N}_{\mu\nu\rho\sigma}(p)(2\pi)^4\delta(p+p'), \quad (14)$$

if the order reduction prescription is employed. It should be emphasized that, contrary to the linearized Einstein tensor $G_{ab}^{(1)}$, the metric perturbation h_{ab} is not gauge invariant. This should not pose a major problem provided that the gauge has been completely fixed, as explained in the literature (Hu *et al.*, 2004; Roura and Verdaguer, manuscript in preparation-a).

The correlation functions in space-time coordinates can be easily derived by Fourier transforming Eq. (13) or (14). There is apparently an infrared divergence at $p^2 = 0$ for the massless case, but such an infrared divergence seems to be just a gauge artifact (Hu *et al.*, 2004). Therefore, we can conclude that, once the instabilities giving rise to the unphysical runaway solutions have been properly dealt with, the fluctuations of the metric perturbations around the Minkowski space-time induced by the interaction with quantum scalar fields are indeed stable (if instabilities had been present, they would have led to a divergent result when Fourier transforming back to space-time coordinates).

5. DISCUSSION

An analysis of the stability of any solution of SCG with respect to small quantum corrections should consider not only the evolution of the expectation value of the metric perturbations around that solution, but also their fluctuations, encoded in the quantum correlation functions. Making use of the equivalence (to leading order in $1/N$, where N is the number of matter fields) between the stochastic correlation functions obtained in stochastic semiclassical gravity and the quantum correlation functions for metric perturbations around a solution of SCG, the symmetrized 2-point quantum correlation function for the metric perturbations can be decomposed into two distinct parts: the intrinsic fluctuations due to the fluctuations of the initial state of the metric perturbations itself, and the fluctuations induced by their interaction with the matter fields. If one considers the linearized perturbations of the semiclassical Einstein equation, only information on the intrinsic fluctuations can be retrieved. On the other hand, the information on the induced fluctuations naturally follows from the solutions of the Einstein–Langevin equation.

As a specific example, we analyzed the symmetrized 2-point quantum correlation function for the metric perturbations around Minkowski space-time interacting

with N scalar fields initially in the Minkowski vacuum state. Once the ultraviolet instabilities which are ubiquitous in SCG (Hu *et al.*, 2004) and commonly regarded as unphysical have been properly dealt with by using the order reduction prescription or the procedure proposed in Hawking *et al.* (2001); Hawking and Hertog (1981), both the intrinsic and the induced contributions to the quantum correlation function for the metric perturbations are found to be stable.

The symmetrized quantum correlation function obtained for the metric perturbations around Minkowski is in agreement with the real part of the propagator obtained by Tomboulis (1977) using a large N expansion (he actually considered fermionic rather than scalar fields, but that just amounts to a change in one coefficient). It is worth noticing that the imaginary part of the propagator can be easily obtained from the expectation value for the commutator of the metric perturbations, which is given by $2i\kappa(G_{\text{ret}}(x', x) - G_{\text{ret}}(x, x'))$, as explained in the literature (Hu *et al.*, 2004; Roura and Verdaguer, manuscript in preparation-a). Tomboulis used the *in-out* formalism rather than the CTP formalism employed in this paper. Nevertheless, his propagator is equivalent to the time-ordered CTP propagator when asymptotic initial conditions are considered because in Minkowski space-time there is no real particle creation and the *in* and *out* vacua are equivalent (up to some phase which is absorbed in the usual normalization of the *in-out* propagator). The use of a CTP formulation is, however, crucial to obtain true correlation functions rather than transition matrix elements in dynamical (nonstationary) situations (such as in an expanding Robertson–Walker background geometry), where the *in-out* scattering matrix might not even be well defined.

It should be mentioned that Ford and collaborators have stressed the importance of the metric fluctuations and investigated some of their physical implications (Borgman and Ford, 2004; Ford, 1982, 1999; Ford and Svaiter, 1997; Ford and Wu, 2003; Ku and Ford, 1993; Yu and Ford, 1999, 2000). They have considered both intrinsic (Ford, 1999; Ford and Svaiter, 1997; Yu and Ford, 1999, 2000) and induced fluctuations (Borgman and Ford, 2004; Ford, 1982, 1999; Ford and Wu, 2003; Ku and Ford, 1993), which they usually refer to as *active* and *passive* fluctuations, respectively. However, they usually consider these two kinds of fluctuations separately and have not provided a unified treatment where both of them can be understood as different contributions to the full quantum correlation function. Moreover, they always neglect the nonlocal term which encodes the averaged back reaction on the metric perturbations due to the modified dynamics of the matter fields generated by the metric perturbations themselves (this term is often called the dissipation term by analogy with quantum Brownian motion models). Their justification is by arguing that those terms would be of higher order in a perturbative expansion. That is indeed the case when considering a Minkowski background if the order reduction prescription is employed, but it is not clear whether it remains true under more general conditions. In fact, as mentioned in Roura and Verdaguer

(manuscript in preparation-b), for the usual cosmological inflationary models the contribution of the nonlocal terms can be comparable or even larger than that of the remaining terms. Finally, to deal with the singular coincidence limit of the noise kernel, Kuo and Ford (1993) opted to subtract a number of terms including the fluctuations for the Minkowski vacuum. Even when no such subtraction was performed (because a method based on multiple integrations by parts was used instead) (Ford, 1999; Wu *et al.*, 2002; Wu and Ford, 2001), they usually discard the fluctuations for the Minkowski vacuum. Therefore, the information on the metric fluctuations around a Minkowski background when the matter fields are in the vacuum state is missing in their work.

An additional number of partially open issues are discussed in Hu *et al.* (2004), to which the reader is referred for further details.

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